MATA SAVITRI DEVI SANJEEVANI PUBLIC SCHOOL

CLASS- XII ELECTROSTATICS SUBJECT: PHYSICS

BY RUPESH GUPTA SIR

We have observed that when two substances are rubbed together, either they starts attracting or repulsing to other substances, then these substances are said to be electrically charged. The charges on insulating bodies cannot move on their own. So it is called static charges.

"The branch of physics, which deals with the study of charges at rest (i.e. static charges), the forces between the static charges, fields and potentials due to these charges, is called electrostatics or static electricity or frictional electricity"

OR

"The branch of science which deals with static or stationary charges is known as electrostatics"

It is impossible to say as to when electricity was first discovered. Around 600 BC by a great philosopher 'Thales of Miletus' (one of the seven wise men of ancient Greece) is said to have observed the attraction of amber, when previously rubbed, for bits of straw. The word electricity has been derived from the Greek word elektron meaning amber.

Sir William Gilbert found that other substances besides amber could also be electrified e.g. glass rod when rubbed with silk. Gilbert classified these materials under two heads Vitreous and Resinous they are named after as positive and negative.

ELECTRIC CHARGE: - According to William Gilbert "Charge is something possessed by material objects that makes it possible for them to exert electrical force and to respond to electrical force."

OR

The additional property of electron, which gives rise to electric force between two electrons is called electric charge.

OR

When two (or more) substances rub together they produce a charge by transfer of electron. This kind of electric charging of an object was called charging by friction.

- Charles Francois Du Fay of France found that there are two kinds of electricity. Benjamin Franklin gave the name: - positive charge and negative charge.
- When a body has excess of electrons it is negatively charged.
- When a body has fewer electrons it is positively charged.
- The property which differentiates the two kinds of charges is called the polarity of charge (i.e. ± sign of charge)
- Gold leaf electroscope (GLE) is the instrument which is used for detecting the presence of electric charge and its polarity.
- Only rubbed area of non conducting body gets charged, and this charge does not move to other parts of the body. The charge is static on rubbed portion only.

ELECTROSTATIC SERIES: - Glass, wool, silk, metal, rubber wax, plastic, sulphur

Electrostatics play a role in Xerox copying machine:- A Xerox copying machine is one of the many industrial applications of the forces of attraction and repulsion between charged bodies. Particles of black powder, called toner stick to a tiny carrier bead of the machine on account of electrostatic forces. The negatively charged toner particles are attracted from carrier bead to a rotating drum, where a positively charged image of document being copied has formed. A charged sheet of paper then attracts the toner particles from the drum to itself. They are then heat fused in place to produce the photocopy.

CONDUCTORS: -A substance which can be used to carry or conduct electricity (or electric charge) from one place to the other is called a conductor.

Silver is one of the best conductors. Copper, iron, aluminium and coal in form of graphite are the conductors. Earth is a good conductor. Human body is also a good conductor of electricity, acids in aqueous form conduct electricity.

In metallic conductors, there are very large number of free electrons which acts as carriers of charges.

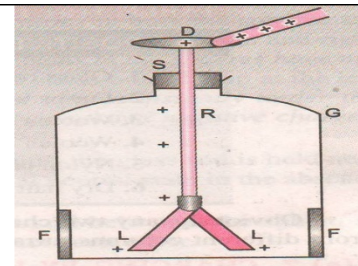
INSULATOR: - the insulators are the materials which cannot conduct electricity i.e. they are poor conductors of electricity.
Glass, rubber, plastic, mica, wax, paper, wood etc. are insulators. As they do not have free electrons so they prevent charge from going one place to another.

DIELECTRICS: - Dielectrics are the insulating materials which transmit electric effects without conducting.

Insulators are also called dielectrics as they cannot conduct electricity however when an external electric field is applied, induced charges appear on the surface of the dielectrics
When a charge body is brought in contact with the earth, all the excess charge on the body disappears by causing a momentary current to pass to the ground through our body. This process of sharing charges with the earth is called grounding or earthing.
Metallic appliances are always to be earthen to prevent them from electrical shock.

Gold Leaf Electroscope (GLE): -it is an instrument which is used for detecting the presence of electric charge and its polarity (i.e. ± Sign of charge). This instrument can also be used for measuring potential difference.
The essential parts of GLE are shown in figure.

- By measuring the divergence of leaves, the amount of charge can be estimated.



ORIGIN OF ELECTRIC CHARGE IN ELECTROSTATICS: - We know that every matter is made of electrons, protons and neutrons (i.e. atom). In atom central core called atomic nucleus, around which negatively charged electrons revolve in circular orbit. Positively charged proton and negatively charged electrons are equal in number and magnitude so whole atom is electrically neutral so we can conclude that the vast amount of charge is an object is usually hidden as the object contains equal amounts of positive charge & negative charge with such an equality or balance of charge, the object is said to be electrically neutral i.e. it contains no net charge. If the positive and negative charges are not in balance, then there is a net charge so to charge a body we remove electron. We can charge a body (i.e. remove electron) by rubbing together. By rubbing we provide energy which is used to remove electrons. Actually electrons are transferred from one body to other so both the bodies get charged.

Further, as an electron has a mass, howsoever small it may be, therefore there does occur some change in mass on charging. A positively charged body has lost some electrons and its mass reduces slightly. On the other hand, a negatively charged body has gained some electrons and hence its mass increases slightly.
A body can be charged by three ways

BY FRICTION
BY INDUCTION
BY CONDUCTION

CHARGING BY FRICTION : Charging by friction

When two bodies are rubbed together, a transfer of electrons takes place from one body to another. The body from which electrons have been transferred is left with an excess of positive charge, so it gets positively charged. The body which receives the electrons becomes negatively charged.

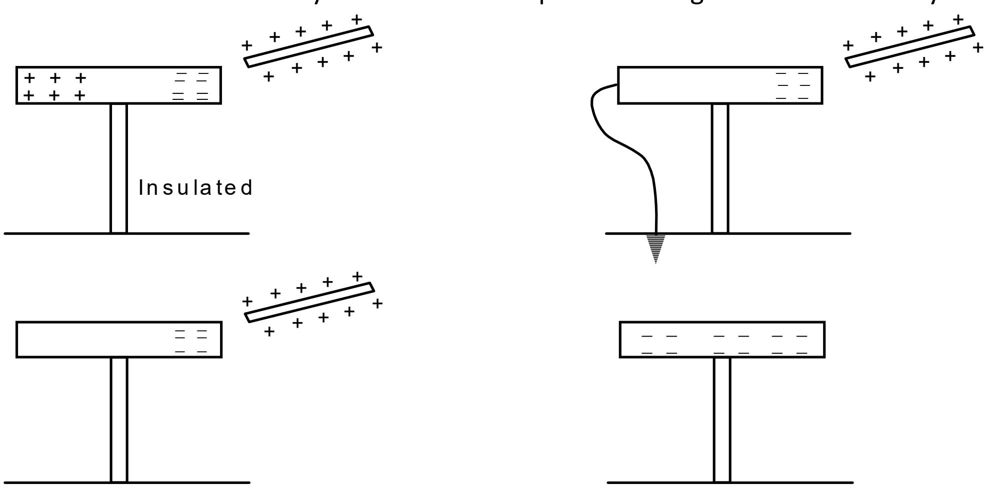
"The positive and negative charges produced by rubbing are always equal in magnitude."

When a glass rod is rubbed with silk, it loses its electrons and gets a positive charge, while the piece of silk acquires equal negative charges.

An ebonite rod acquires a negative charge, if it is rubbed with wool (or fur). The piece of wool (or fur) acquires an equal positive charge.

CHARGING BY INDUCTION: - A charge body A imparts to another body B, some charge of opposite sign without any actual contact between A and B.

If a positively charged rod is brought near an insulated conductor, the negative charges (electrons) in the conductor will be attracted towards the rod. As a result, there will be an excess of negative charge at the end of the conductor near the rod and the excess of positive charge at the far end. This is known as 'electrostatic induction'. The charges thus induced are found to be equal and opposite to each other. Now if we touch the far end with a conductor connected to the earth, the positive charges here will be cancelled by negative charges coming from the earth through the conducting wire. Now, if we remove the wire first and then the rod, the induced negative charges which were held at the outer end will spread over the entire conductor. It means that the conductor has become negatively charged by induction. In the same way one can induce a positive charge on a conductor by bringing a negative charged rod near it.



Important points regarding electrostatic induction

(a) Inducing body neither gains nor loses charges.

(b) The nature of induced charge is always opposite to that of inducing charge.

(c) Induced charge can be lesser or equal to inducing charge but it is never greater than the inducing charge.

(d) Induction takes place only in bodies (either conducting or nor conducting) and not in particles.

CHARGING BY CONDUCTION: - Charging by actual contact.

Let us consider two conductors, one charged and the other uncharged. We bring the conductors in contact with each other. The charge (whether negative or positive) under its own repulsion will spread over both the conductors. Thus the conductors will be charged with the same sign. This is called 'charging by conduction (through contact)'.

NOTE: - The process of electric induction, the positively charged glass rod does not lose any charge where as in conduction process body loses some charge.

PROPERTIES OF ELECTRIC CHARGES

- Charge is always associated with mass. The charge cannot exist without mass though mass can exist without charge.
- Opposite charges (or unlike charges) attract each other and similar charges (like charges) repel each other.
- Electric charge is conserved in nature (i.e. charge neither be created nor be destroyed)

  OR conservation of charge is the property by virtue of which total charge of an isolated system always remain constant or conserved.
- Electric charge is additive (i.e. total charge is the algebraic sum of the individual charge).

If a system contain n charges $q_1, q_2, q_3, \ldots\ldots\ldots\ldots q_n$. then the total charge of the system is

$q = q_1 + q_2 + q_3 + \ldots\ldots\ldots\ldots + q_n$.

- Electric charge is quantized and the quantum of charge is equal to that of one electron.

The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge of an electron/proton, represented by e

$q = ne$

Here $e = 1.6 \times 10^{-19}$ C

- **Charge is invariant**
- This means that charge is independent of frame of reference, i.e., charge on a body does not change whatever be its speed.

COMPARISION OF CHARGE AND MASS

CHARGE	MASS
1. Electric charge on a body may be positive, negative or zero	1. Mass of a body is positive quantity.
2. Chare carried by a body does not depend upon velocity of the body	2. Mass of the body increase with its velocity $m = \frac{m_0}{\sqrt{1-v^2/c^2}}$ Where c is velocity of light in vacuum, m is the mass of the body moving with velocity v and m_0 is the rest mass of the body.
3. Charge is quantized	3. Quantization of mass is yet to be established.
4. Electric charge is conserved	4. Mass is not conserved.
5. Force between charges can be attractive or repulsive.	5. The gravitational force is always attractive.

COULOMB'S LAW: - the force of interaction between any two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them

$$F \propto \frac{|q_1|\,|q_2|}{r^2}$$

or
$$F = K\frac{|q_1|\,|q_2|}{r^2} \qquad \text{---(1)}$$

Where K is electrostatic force constant.

The value of electrostatic force constant K depends on the nature of medium separating the charges, and on the system of units.

When the charges are situated in free space (air/vacuum) then in CGS system K =1

In SI $K = 9 \times 10^9\ Nm^2C^{-2}$

We can write $K = 1/4\pi\varepsilon_0$ -- (2)

Where ε_0 is called absolute electrical permittivity of the free space

Therefore, from equation (1), the magnitude of force is

$$F = 1/4\pi\varepsilon_0 \frac{|q_1|\,|q_2|}{r^2} \qquad \text{---(3)}$$

Unit of ε_0 $C^2N^{-1}m^{-2}$

Dimensions of ε_0 $[M^{-1}L^{-3}T^4A^2]$

Value of ε_0 $8.85 \times 10^{-12}\ C^2N^{-1}m^{-2}$

COULOMB'S LAW IN VECTOR FORM

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2} \times \hat{r}_{21} \quad \text{or} \quad \vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}^2} \times \hat{r}_{12}$$

$$\vec{F}_{12} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{|r_{12}|^3} \cdot \vec{r}_{21}$$

$$\vec{F}_{21} = \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3}$$

Note :- $|r_{12}| = |r_{21}|$

$$\vec{F}_{12} = -\vec{F}_{21}$$

UNIT OF CHARGE: - unit of charge in SI is coulomb (C)

One Coulomb is that much charge which when placed in vacuum at a distance of one metre from an equal and similar charge would repel it with a force of $9X10^9$ Newton.

The CGS unit of charge is 1 electrostatic unit (e.s.u) of charge or stat coulomb or franklin

Charge on an electron = $1.6X10^{-19}$ C = 4.8 X 10^{-10} stat coulomb

Therefore, 1 Coulomb = 3 X 10^9 stat Coulomb.

Another unit of charge is electromagnetic unit (e.m.u) of charge

1 e.m.u of charge = 3 X 10^{10} e.s.u of charge (State Coulomb)

= 10 Coulomb.

DIELECTRIC CONSTANT OR RELATIVE ELECTRICAL PERMITTIVITY

Dielectric constant of a medium is the ratio of absolute electrical permittivity of the medium to the absolute electrical permittivity of free space.

Suppose charges are situated in medium, the force between them is

$$F_m = \frac{q_1 q_2}{4\pi\varepsilon_m r^2} \quad \text{--(1)}$$

Where ε_m is called absolute electrical permittivity of the intervening medium.

The force between the same two charges held in free space

$$F_o = \frac{q_1 q_2}{4\pi\varepsilon_o r^2} \quad \text{--(2)}$$

Dividing (2) by (1)

We have $\frac{F_o}{F_m} = \frac{\varepsilon_m}{\varepsilon_o} = \varepsilon_r$ or K

ε_r is called relative electrical permittivity of the medium.

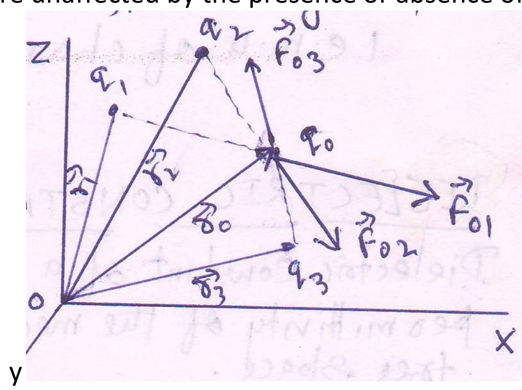
It is also called dielectric constant of the medium and denoted by K. the value of K depends only on the nature of medium. For vacuum K =1; for air K =1.006; for hydrogen K = 1.00026

For Gas K = 3 to 4; for mica = 3 to 6; for water K = 81.

FORCES BETWEEN MULTIPLE CHARGES (PRINCIPLE OF SUPERPOSITION)

According to superposition principle, total force on any charge due to a number of other charges at rest is the vector sum of all the forces on that charge due to other charges, taken one at a time.

The forces due to individual charges are unaffected by the presence or absence of other charges.



Suppose charges q_1, q_2, q_3……….. q_n are situated at points with position vectors $\mathbf{r_1}$, $\mathbf{r_2}$, $\mathbf{r_3}$, ………… $\mathbf{r_n}$ respectively w.r.t the origin O in system xyz.

Total force $\mathbf{F_0}$ on a test charge q_0 at position $\mathbf{r_0}$ due to all n discrete charges

$$\mathbf{F} = \mathbf{F_{01}} + \mathbf{F_{02}} + \mathbf{F_{03}} + \ldots\ldots\ldots + \mathbf{F_{0n}} \quad \text{---------------------------------(1)}$$

Here $\mathbf{F_{01}}$ force on q_0 due to q_1; $\mathbf{F_{02}}$ force on q_0 due to q_2 & so on.

According to coulomb's law

$$\vec{F}_{01} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q_1}{r_{10}^2}\hat{r}_{10}$$

$$\vec{F}_{02} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q_2}{r_{20}^2}\hat{r}_{20}$$

$$\vdots$$

$$\vec{F}_{0n} = \frac{1}{4\pi\varepsilon_0}\frac{q_0 q_n}{r_{n0}^2}\hat{r}_{n0}.$$

Putting in eq (1) we get

$$\vec{F}_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_0 q_1}{r_{10}^2}\hat{r}_{10} + \frac{q_0 q_2}{r_{20}^2}\hat{r}_{20} + \cdots + \frac{q_0 q_n}{r_{n0}^2}\hat{r}_{n0}\right]$$

$$\vec{F}_0 = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{i=n} \frac{q_0 q_i}{r_{i0}^2} \hat{r}_{i0}$$

$$\text{or} \quad \vec{F}_0 = \frac{1}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_0 q_i}{r_{i0}^3} \vec{r}_{i0}$$

$$\text{or} \quad \vec{F}_0 = \frac{q_0}{4\pi\varepsilon_0} \sum_{i=1}^{n} \frac{q_i}{|\vec{r}_0 - \vec{r}_i|^3} (\vec{r}_0 - \vec{r}_i)$$

SURFACE CHARGE DENSITY (σ)

$\sigma = \Delta Q/\Delta s$

Here ΔQ is the amount on element and Δs is the surface area of conductor

Unit: - C/m^2

LINEAR CHARGE DENSITY (λ)

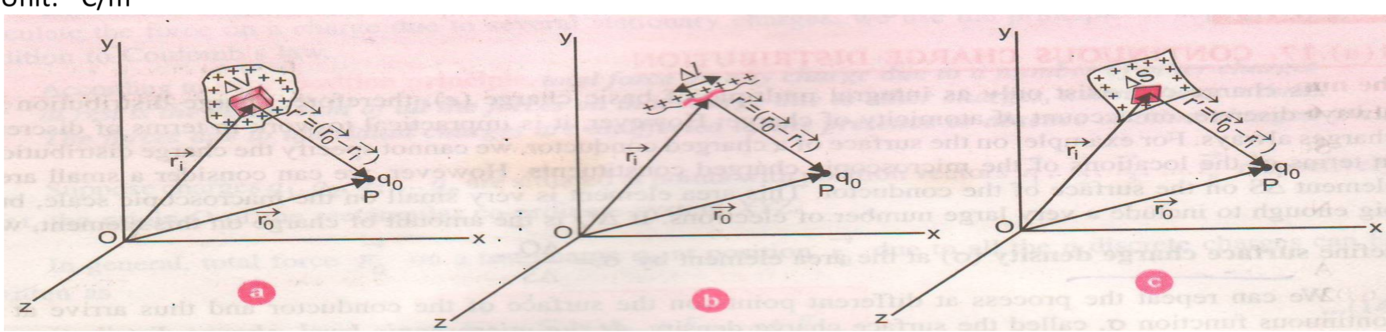
$\lambda = \Delta Q/\Delta l$

Unit: - C/m

VOLUME CHARGE DENSITY (ρ)

$\rho = \Delta Q/\Delta V$

Unit: - C/m^3



By the superposition principle, total force due to entire volume charge distribution is obtained by

$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0} \sum_{\text{all}\,\Delta V} \frac{\rho \Delta V}{r'^2} \hat{r}'$$

when $\Delta V \to 0$, the sum becomes an integral & total force

$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0} \int_V \frac{\rho \Delta V}{r'^2} \hat{r}'$$

similarly

$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0} \int_L \frac{\lambda \Delta L}{r'^2} \hat{r}'$$

$$\vec{F} = \frac{q_0}{4\pi\varepsilon_0} \int_S \frac{\sigma\, ds}{r'^2} \hat{r}'$$

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Illustration 1

Question: **How many electrons must be removed from a piece of metal so as to leave it with a positive charge of 3.2×10^{-17} coulomb?**

Solution: From 'Quantization of charge', we know $Q = ne$

$$\therefore n = \frac{Q}{e} = \frac{3.2\times10^{-17}C}{1.6\times10^{-19}C} = \mathbf{200}$$

Illustration 2

Question: **A copper penny has a mass of 32 g. Being electrically neutral, it contains equal amounts of positive and negative charges. What is the magnitude of these charges inμC. A copper atom has a positive nuclear charge of 3×10^{-26} C. Atomic weight of copper is 64g/mole and Avogadro's number is 6×10^{-26} atoms/mole.**

Solution: 1 mole i.e., 64 g of copper has 6×10^{23} atoms. Therefore, the number of atoms in copper penny of 32 g is

$$\frac{6\times10^{23}}{64}\times32\times10^{-3} = 3 \times 10^{20}$$

One atom of copper has each positive and negative charge of 3×10^{-26} C. So each charge on the penny is $(3 \times 10^{20}) \times (3 \times 10^{-26})$ = **9μC.**

Illustration 3

Question: **The electron and the proton in a hydrogen atom are 0.53×10^{-11} m apart. Compare the electrostatic and the gravitational forces between them in power of 10^{-41}.**

Solution: The magnitude of the electrostatic force is

$$F_E = \frac{1}{4\pi\varepsilon_0}\frac{e^2}{r^2}$$

$$= \frac{(9\times10^9\ \mathrm{N-m^2/C^2})\times(1.6\times10^{-19}\mathrm{C})^2}{(5.3\times10^{-11}\mathrm{m})^2}$$

$$= 8.2 \times 10^{-8}\ \mathrm{N}$$

The magnitude of the gravitational force is

$$F_G = G\frac{m_e m_p}{r^2}$$

$$= \frac{(6.67\times10^{-11}\mathrm{N-m^2/kg^2})(9.11\times10^{-31}\mathrm{kg})(1.67\times10^{-27}\mathrm{kg})}{(5.3\times10^{-11}\mathrm{m})^2}$$

$$= 3.6 \times 10^{-47}\ \mathrm{N}$$

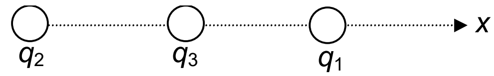
The ratio of the forces

$$\frac{F_G}{F_E} = \mathbf{44}$$

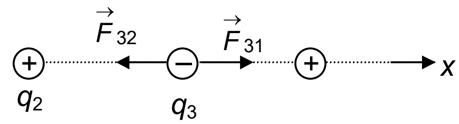
☞ The $\frac{F_G}{F_E}$ is extremely small. So when we deal with the electrical interaction between elementary particles, gravity may safely be ignored.

Illustration 4

Question: **Three charges lie along the *x*-axis as shown in the figure. The positive charge q_1 = 15.0 μC is at x = 2.0 m, and the positive charge q_2 = 6.00 μC is at the origin. Where must a negative charge q_3 be placed on the x-axis such that the resultant force on it is zero? (in mm)**



Solution: Since q_3 is negative and both q_1 and q_2 are positive, the forces $\vec{F}_{31}$ and $\vec{F}_{32}$ are both attractive. Let x be the co-ordinate of q_3



We have

$$F_{31} = \frac{K|q_3|\ |q_1|}{(2-x)^2};\ F_{32} = \frac{K|q_3|\ |q_2|}{x^2}$$

Since the net force on the change q_3 is zero,

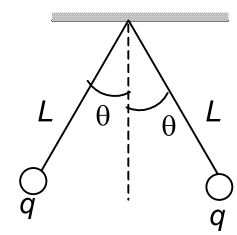
we have, $\frac{K|q_3|\ |q_2|}{x^2} = \frac{K|q_3|\ |q_1|}{(2-x)^2}$

or, $(4 - 4x + x^2)\,(6 \times 10^{-6}\text{C}) = x^2\,(15 \times 10^{-6}\text{ C})$

Solving this quadratic equation for x, we get x = **775**

Illustration 5

Question: **Two identical small charged sphere, each having a mass of 3.0 × 10^{-2} kg, hang in equilibrium as shown below. If the length of each string is $\sqrt{\frac{3}{2}}$ m and the angle θ = 45°, find the magnitude of the charge on each sphere in nC. (g = 10 m/s^2)**

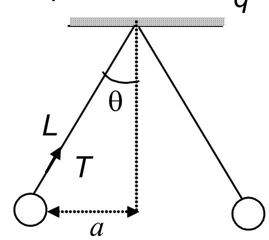


Solution: From the right angled triangle, we have $\sin\theta = \frac{a}{L}$

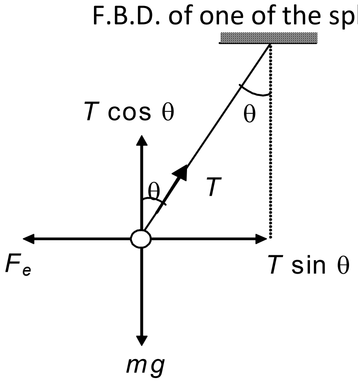
or, $a = L\sin\theta$ = (15 m) sin 5° = 0.013 m

Hence, the separation of the spheres is

$2a$ = 0.026 m



F.B.D. of one of the spheres:-



Since the sphere is in equilibrium, the resultants of the forces in the horizontal and vertical directions must separately add up to zero. thus

$T\sin\theta - F_e = 0$

$\Rightarrow$ $T \sin\theta = F_e$ ... (i)

and $T \cos\theta - mg = 0$

$\Rightarrow$ $T \cos\theta = mg$ ... (ii)

Dividing equation (i) by equation (ii) , we get

$$\tan\theta = \frac{F_e}{mg} \quad \text{or,} \quad F_e = mg \tan\theta$$

$$= (3 \times 10^{-2}\ \text{kg}) \times (10\ \text{m/s}^2)(\tan 45°)$$

$$= 0.3\ \text{N}$$

Let q be charge on each sphere.

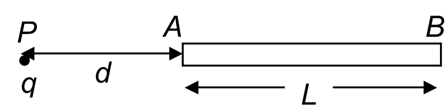
According to Coulomb's law

$$F_e = \left(\frac{1}{4\pi\varepsilon_0}\right)\frac{|q||q|}{r^2}$$

$\therefore$ q = **15 μC**

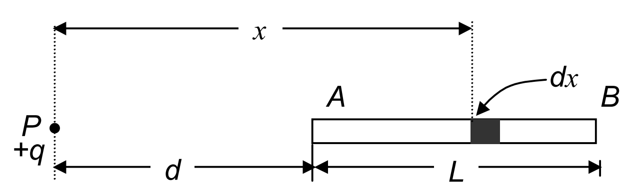
Illustration 6

Question: **A point charge *q* is situated at a distance '*d*' from one end of a thin non-conducting rod of length *L* having a charge *Q* (uniformly distributed along its length) as shown. Find the magnitude of the electric force between the two. (in N)**



(take $q = 10^{-6}$C, $Q = 9 \times 10^{-3}$C, d = 1m, L = 2m)

Solution: Consider an element of rod of length 'dx' at a distance x from the point charge q. Treating the element as a point charge, the force between q and the charge element will be



$$dF = \frac{1}{4\pi\varepsilon_0}\frac{q\,dQ}{x^2}$$

But $$dQ = \left(\frac{Q}{L}\right)dx$$

So, $$dF = \frac{1}{4\pi\varepsilon_0}$$

$$\frac{qQdx}{Lx^2}$$

$$\therefore \quad F = \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\int_d^{(d+L)}\frac{dx}{x^2}$$

$$= \frac{1}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_d^{d+L}$$

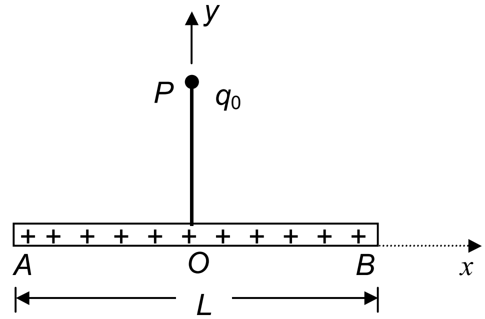
$$= \frac{1}{4\pi\varepsilon_0}\frac{qQ}{L}\left[\frac{1}{d}-\frac{1}{d+L}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{qQ}{d(d+L)}$$

$$= \mathbf{27}$$

Illustration 7

Question: **A thin non-conducting rod of length *L* and having a charge *Q* (uniformly distributed along its length) is placed along *x*-axis, as shown. Find the force (in N) exerted by the rod on the point charge q_0 located on the perpendicular bisector of the rod (the positive *y*-axis) at a distance *y* from the centre (take *y* =1m, L = 2m, $q_0 = \sqrt{2} \times 10^{-6}$C, q= 9×10^{-3} C.)**



Solution: Consider an element of rod of length 'dx' at a distance x from the centre. Treating the element as a point charge, the force between *q* and the charge element will be

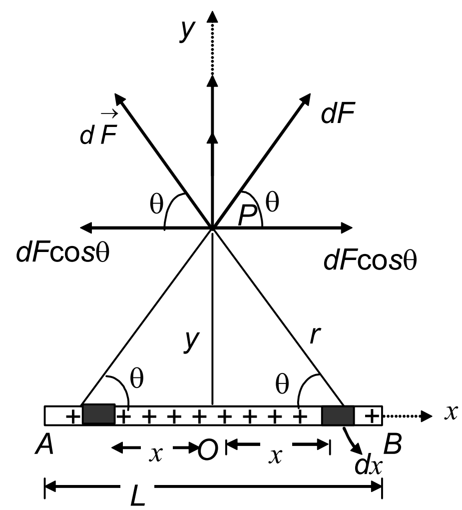
$$dF = \frac{1}{4\pi\varepsilon_0}\frac{q_0\,dq}{r^2}$$

The direction of $d\vec{F}$ is shown in the figure.

But $dq = \lambda dx$

So, $$dF = \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda dx}{r^2}$$

For every charge element *dq* located at position $+x$, there is another charge element *dq* located at $-x$. When we add the forces due to the charge elements at $+x$ and $-x$, we find the x components have equal magnitudes but point in opposite directions. So their sum is zero, i.e., F_x = 0



Now, $dF_y = dF \sin\theta$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda dx}{x^2+y^2}\frac{y}{\sqrt{x^2+y^2}}$$

($\therefore r^2 = x^2 + y^2$; $\sin\theta = \frac{y}{\sqrt{x^2+y^2}}$)

or, $$dF_y = \frac{1}{4\pi\varepsilon_0}\frac{q_0\lambda y dx}{(x^2+y^2)^{3/2}}$$

$$\therefore \quad F_y = \int dF_y = \frac{1}{4\pi\varepsilon_0} q_0\lambda\, y \int_{-\frac{L}{2}}^{+\frac{L}{2}} \frac{dx}{(x^2+y^2)^{3/2}}$$

Evaluating the integral, we obtain

$$F_y = \frac{1}{4\pi\varepsilon_0} \frac{q_0 q}{y\sqrt{y^2 + \frac{L^2}{4}}} = \mathbf{81}$$

ELECTRIC FIELD: - Electric field due to a given charge as the space around the charge in which electrostatic force of attraction or repulsion due to the charge can be experienced by any other charge.

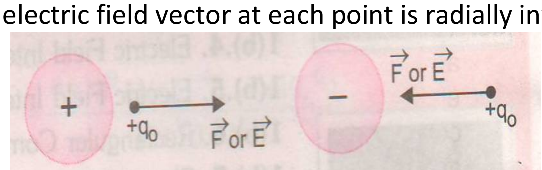
ELECTRIC FIELD INTENSITY: - the electric field intensity at any point is the strength of electric field at that point. It is defined as the force experienced by unit positive charge placed at that point.

E(r) = F(r)/q_0

The SI Unit of electric field intensity is N/C

Electric field intensity is a vector quantity.

- The direction of the electric field intensity at a point inside the electric field is the direction in which the electric field exerts force on a (unit) positive charge.
- For a positive source charge, the electric field will be directed radially outwards from the charge. If the source charge is negative, the electric field vector at each point is radially inwards.



- As the magnitude of the test charge q_0 decreases, electric field intensity **E(r)** at a point is defined more and more accurately. On account of discrete nature of charge, the minimum possible value of test charge q_0 is $1.6\text{X}10^{-19}$ C. it cannot be zero
- If the small test charge q_0 is positive, the measured value of electric intensity will be somewhat less than the actual value of electric intensity. However, if the small test charge q_0 is negative, the measured value of electric intensity will be somewhat more than the actual value of electric intensity.
- Dimensions of the electric field intensity

$$E = \frac{F}{q_0} = \frac{[MLT^{-2}]}{[AT]} = [MLT^{-3}A^{-1}]$$

In S.I. systems, the unit of $\vec{E}$ is N/C or V/m as

$$\frac{\text{N}}{\text{C}} = \frac{\text{N}\times\text{m}}{\text{C}\times\text{m}} = \frac{\text{J}}{\text{C}\times\text{m}} = \frac{\text{V}}{\text{m}}$$

FORCED EXERTED BY A FIELD ON A CHARGE INSIDE IT

By definition as $\vec{E} = \frac{\vec{F}}{q_0}$, i.e.,

$$\vec{F} = q_0 \vec{E}$$

If q_0 is a +ve charge, force $\vec{F}$ on it is in the direction of $\vec{E}$.

If q_0 is a –ve charge, $\vec{F}$ on it is opposite to the direction of $\vec{E}$

Illustration 8

Question: **An electron ($q = -e$) is placed near a charged body experiences a force in the positive y direction of magnitude 3.60×10^{-8} N.**

(a) The electric field at that location is $x \times 10^{-9,}$ find x. (where x is in N)

(b) What would be the force exerted by the same charged body on an alpha particle ($q = +2e$) placed at the location initially occupied by the electron?

Solution: Using equation (7), we have

$$E_y = \frac{|F_y|}{|q_0|} = \frac{3.60 \times 10^{-8}\text{N}}{1.60 \times 10^{-19}\text{C}} = \mathbf{225}$$

The electric field is in the negative y direction.

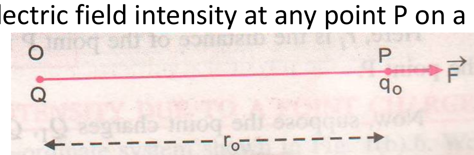
(b) The force on the alpha particle is given by

$F_y = q_0 E_y = 2(+1.60 \times 10^{-19}\text{ C})(2.25 \times 10^{11}\text{ N/C}) = \mathbf{72}$

The force is in the negative y direction, the same direction as the electric field.

ELECTRIC FIELD INTENSITY DUE TO A POINT CHARGE

Suppose we have to calculate electric field intensity at any point P on a small positive charge q_0



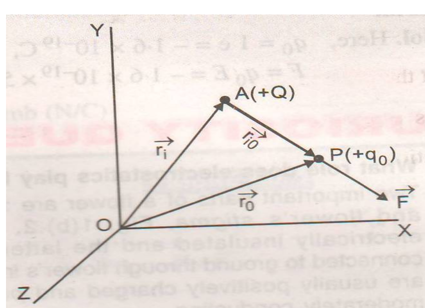
By coulomb's law, force on charge q_0 at P

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r_0^2}\hat{r}_0$$

$$\text{As } \vec{E} = \frac{\vec{F}}{q_0}.$$

$$\therefore \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r_0^2}\hat{r}_0 \quad \text{———} \quad (1)$$

In XYZ coordinate system



According to coulomb's law, force on a small test charge (+q_0) at P is

$$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r_{10}^2}\hat{r}_{10} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{r_{10}^3}\vec{r}_{10}$$

OR $$\vec{F} = \frac{1}{4\pi\varepsilon_0}\frac{Qq_0}{|\vec{r}_0-\vec{r}_1|^3}(\vec{r}_0-\vec{r}_1)$$

As $$\vec{E} = \frac{\vec{F}}{q_0}$$

$$\therefore \vec{E} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{|\vec{r}_0-\vec{r}_1|^3}(\vec{r}_0-\vec{r}_1) \quad \text{———} (2).$$

As $\vec{r}_0 - \vec{r}_1 = \vec{r}_{10} = \vec{AP}$

obviously, it is along AP produced.

ELECTRIC FIELD INTENSITY DUE TO A GROUP CHARGES

$$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3 + \cdots + \vec{E}_n$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=n}\frac{Q_i}{r_i^2}\hat{r}_i$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{i=n}\frac{Q_i}{|\vec{r}_0-\vec{r}_i|^3}(\vec{r}_0-\vec{r}_i).$$

ELECTRIC FIELD INTENSITY DUE TO CONTINUOUS CHARGE DISTRIBUTION

- Electric field intensity at location $\vec{r}_0$ due to linear charge distribution $\lambda(\vec{r}_i)$ is

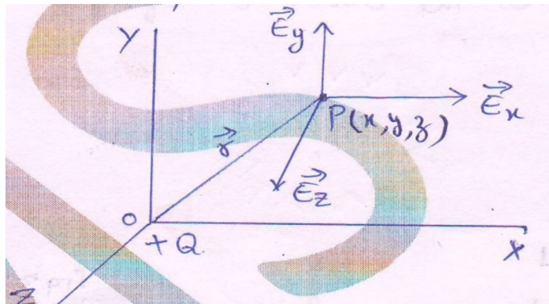
$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_L \frac{\lambda(\vec{r}_i)\,dl}{|\vec{r}_0-\vec{r}_i|^3}(\vec{r}_0-\vec{r}_i)$$

Electric field intensity at location $\vec{r}_0$ due to surface charge distribution $\sigma(\vec{r}_i)$ and volume charge distribution $\rho(\vec{r}_i)$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_S \frac{\sigma(\vec{r}_i)\,dS}{|\vec{r}_0-\vec{r}_i|^3}(\vec{r}_0-\vec{r}_i)$$

$$\vec{E} = \frac{1}{4\pi\varepsilon_0}\int_V \frac{\rho(\vec{r}_i)\,dV}{|\vec{r}_0-\vec{r}_i|^3}(\vec{r}_0-\vec{r}_i).$$

RECTANGULAR COMPONENTS OF ELECTRIC INTENSITY DUE TO A POINT CHARGE: - suppose a point charge +Q is held at O, the origin of co-ordinate system. We have to calculate rectangular components of electric field intensity (**E**) at any point P(x, y, z)



$$\vec{OP} = \vec{r} = \hat{i}x + \hat{j}y + \hat{k}z$$

$$r = |\vec{r}| = \sqrt{x^2 + y^2 + z^2}$$

force on unit +ve charge at P.

$$\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{Q.(1)}{r^2} \hat{r}$$

Electric field intensity on unit +ve charge at P.

$$\vec{E} = \frac{1}{4\pi\varepsilon_0} \frac{Q}{r^2} \hat{r}$$

$$\vec{E} = \frac{Q}{4\pi\varepsilon_0} \frac{\vec{r}}{r^3} = \frac{Q.(\hat{i}x + \hat{j}y + \hat{k}z)}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{3/2}} \quad \text{------- (1)}$$

If $\vec{E}_x$, $\vec{E}_y$, $\vec{E}_z$ are the components of $\vec{E}$ along the three co-ordinate axes, then

$$\vec{E} = \vec{E}_x + \vec{E}_y + \vec{E}_z$$

$$\text{or } \vec{E} = \hat{i}E_x + \hat{j}E_y + \hat{k}E_z \quad \text{------- (2)}$$

comparing eq (1) and (2), we get

$$E_x = \frac{Qx}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{3/2}}; \quad E_y = \frac{Qy}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{3/2}}$$

$$E_z = \frac{Qz}{4\pi\varepsilon_0 (x^2 + y^2 + z^2)^{3/2}}$$

PHYSICAL SIGNIFICANCE OF ELECTRIC FIELD: -from the knowledge of electric field intensity **E** at any point **r**, we can readily calculate the magnitude and direction of force experienced by any charge q_0 held at that point i.e.

$$\vec{F}(\vec{r}) = q_0 \vec{E}(\vec{r})$$

MATA SAVITRI DEVI SANJEEVANI PUBLIC SCHOOL

CLASS- XII ELECTROSTATICS SUBJECT: PHYSICS

BY RUPESH GUPTA SIR

ELECTRIC FIELD LINES: - an electric field lines as a path, straight or curved in electric field, such that tangent to it at any point gives the direction of electric field intensity at that point.

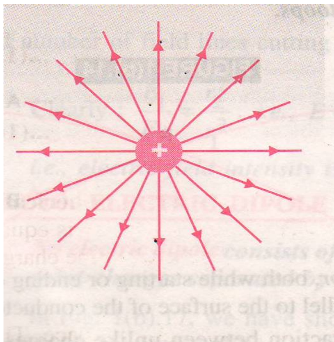


Figure 1

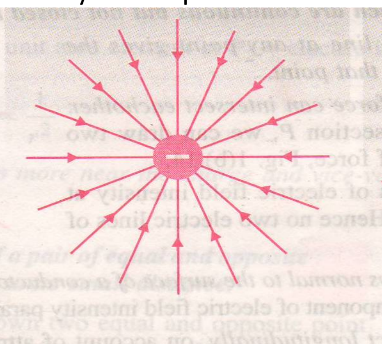


Figure 2

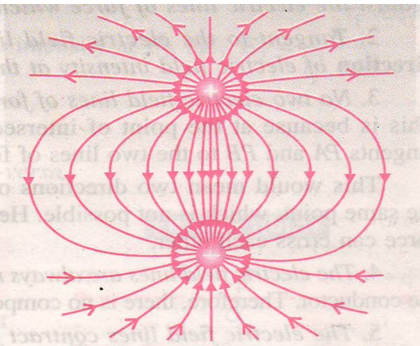


Figure 3

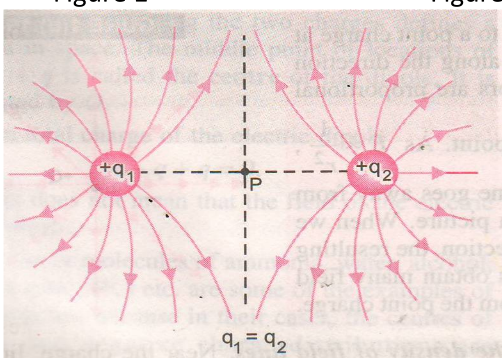


Figure 4

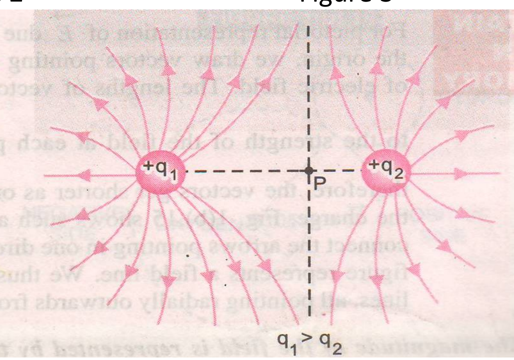
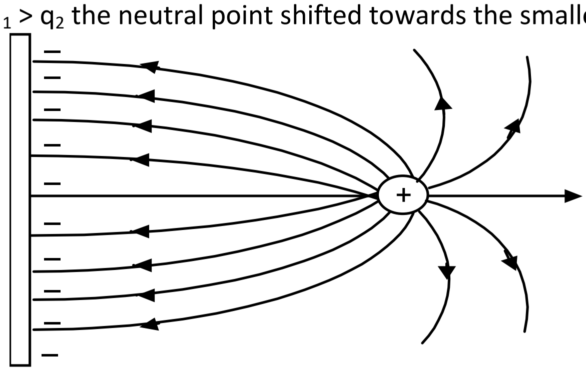


Figure 5

Figure 1 shows some lines of force due to single positive charge. These are directed radially outwards. Figure 2 shows lines of force due to single negative point charge these are directed radially inwards. Figure 3 shows lines of force due to a pair of equal and opposite charges (q, -q) forming an electric dipole. In Figure 4 charge are equal their neutral point P lies in the centre. In Figure 5 when $q_1 > q_2$ the neutral point shifted towards the smaller charge q_2.



Fixed point charge near infinite metal plate

PROPERTIES OF ELECTRIC FIELD LINES: -

1. Electric field lines are continuous curve. They start from a positively charged body and end at a negatively charged body.
2. No electric lines of force exist inside the charged body, thus electrostatic field lines do not form continuous closed loops.

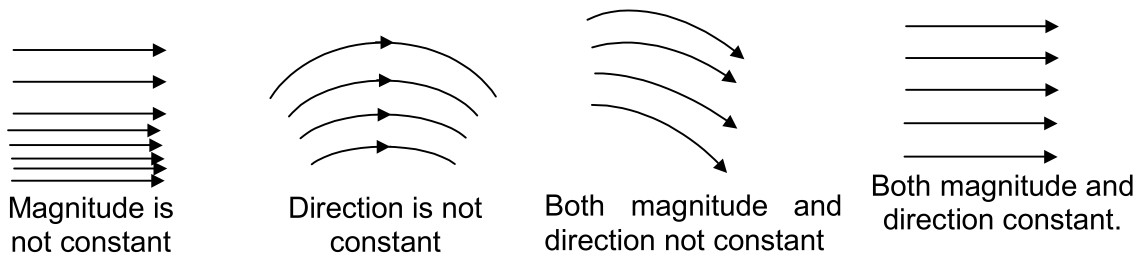
- If there is a single charge, the electric field lines may start or end at infinity, due to a single negative charge, field lines would start from infinity and end at the negative charge.

3. Tangent to the electric field line at any point gives the direction of electric field intensity at that point.
4. No two electric field lines of force can intersect each other.
5. The electric field lines are always normal to the surface of a conductor.
6. The electric field lines contract longitudinally.
7. The electric field lines exert a lateral pressure.

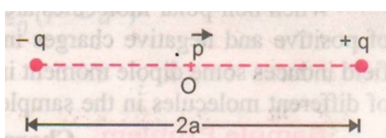
- The magnitude of the field is represented by the density of field lines. So we can defined electric field intensity as

"Electric field intensity at a point is equal to number of field lines crossing normally a unit area around that point"

8. The number of electric lines of force that originate from or terminate on a charge is proportional to the magnitude of the charge.
9. As number of lines of force per unit area normal to the area at point represents magnitude of intensity, crowded lines represent strong field while distant lines weak field. Further, if the lines of force are equidistant straight lines, the filed is uniform.



ELECTRIC DIPOLE: -An electric dipole consists of a pair of equal and opposite point charges separated by some small distance.



We have shown two equal and opposite point charges (±q) separated by a small distance '2a'. it represents an electric dipole. 2a is called length of dipole.
The total charge of the electric dipole = -q +q = 0

- This does not mean that the field of the electric dipole is zero.

DIPOLE MOMENT: - dipole moment (**p**) is a measure of the strength of electric dipole. It is a vector quantity whose magnitude is equal to product of magnitude of either charge and distance between them

i.e. $\mathbf{p} = q\,(2\mathbf{a})$

or $|\mathbf{p}| = q(2a)$

by convention, the direction of **p** is from negative charge to positive charge.
SI unit C-m

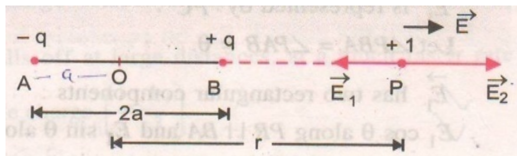
- If charge q gets larger and 2a gets smaller and smaller, keeping the product $|\mathbf{p}|$= qx2a constant we get an ideal dipole or point dipole.

PHYSICAL SIGNIFICANCE OF ELECTRIC DIPOLES: - the study of electric dipoles is important for electrical phenomana in matter.

- If the centre of mass of positive charges coincide with the centre of mass of negative charges, the molecule behave as a non-polar molecule. On the contrary, if the centre of mass of positive charges does not coincide with the centre of mass of negative charges, the molecule behaves as a polar molecule.

DIPOLE FILED: - the space around the dipole in which the electric effect of the dipole can be experienced is called dipole field.

FIELD INTENSITY ON AXIAL LINE OF ELECTRIC DIPOLE: - Consider an electric dipole consisting of two point charges –q and +q separated by a small distance 2a. We have calculated electric intensity **E** at a point P. on the axial line of the dipole and at a distance r from a centre O.



$\mathbf{E_1}$ is the electric intensity at P due to charge –q at A

$$|\vec{E_1}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

It is along PA.

$\vec{E_2}$ is the electric intensity at P due to charge q at B.

$$|\vec{E_2}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

It is along BP

$\vec{E_2}$ is the electric intensity at P due to charge q at B.

$$|\vec{E_2}| = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2}$$

It is along BP

clearly $|\vec{E_2}| > |\vec{E_1}|$

the resultant intensity

$$|\vec{E}| = |\vec{E_2}| - |\vec{E_1}|$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{(r-a)^2} - \frac{1}{4\pi\varepsilon_0}\frac{q}{(r+a)^2}$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{(r-a)^2} - \frac{1}{(r+a)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{(r+a)^2 - (r-a)^2}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\left[\frac{\cancel{r^2}+\cancel{a^2}+2ra-\cancel{r^2}-\cancel{a^2}+2ra}{(r^2-a^2)^2}\right]$$

$$= \frac{q}{4\pi\varepsilon_0}\frac{4ra}{(r^2-a^2)^2}$$

$$= \frac{q(2a)\,2r}{4\pi\varepsilon_0(r^2-a^2)^2}$$

$$|\vec{E}| = \frac{|\vec{p}|\,2r}{4\pi\varepsilon_0(r^2-a^2)^2}$$

If dipole is short i.e $2a << r$

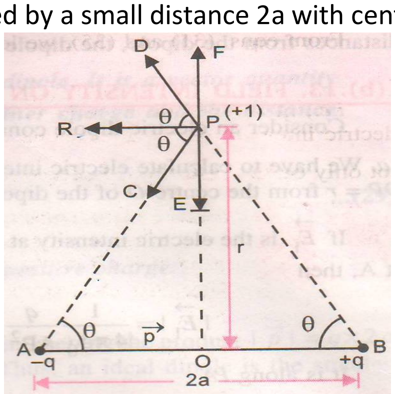
$$|\vec{E}| = \frac{|\vec{p}|\,2r}{4\pi\varepsilon_0 r^4}$$

$$= \frac{2|\vec{p}|}{4\pi\varepsilon_0 r^3}$$

The direction of $\vec{E}$ is along BP produced.

clearly $|\vec{E}| \propto 1/r^3$

FIELD INTENSITY ON EQUATORIAL LINE OF ELECTRIC DIPOLE: - consider an electric dipole consisting of two point charges –q and +q separated by a small distance 2a with centre O and dipole moment p = q(2a).



We have to find electric intensity **E** at a point P on the equatorial line of the dipole, OP = r

E_1 is the electric intensity at P due to charge $-q$ at A,

then $|\vec{E}_1| = \frac{1}{4\pi\varepsilon_0}\frac{q}{AP^2}$.

But $AP^2 = OP^2 + OA^2$

$= r^2 + a^2$

$\therefore \quad |\vec{E}_1| = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2}$ ——— (1)

$\vec{E}_1$ is represented by $\vec{PC}$

let $\angle PBA = \angle PAB = \theta$.

$\vec{E}_1$ has two rectangular components

$E_1\cos\theta$ along $PR \parallel BA$

and $E_1\sin\theta$ along $PE \perp BA$.

E_2 is electric intensity at P, due to charge $+q$ at B, then

$|\vec{E}_2| = \frac{1}{4\pi\varepsilon_0}\frac{q}{BP^2}$

$= \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2+a^2}$ $\quad ; \quad BP^2 = r^2 + a^2$. ——— (2)

$\vec{E}_2$ is represented by $\vec{PD}$ (along BPD)

$\vec{E}_2$ has two rectangular components

$E_2\cos\theta$ along $PR \parallel BA$

and $E_2\sin\theta$ along PF (opposite to PE)

from eq (1) and (2)

$|\vec{E}_1| = |\vec{E}_2|$

$\therefore \quad E_1\sin\theta$ along PE and $E_2\sin\theta$ along PF cancel out

$\therefore$ Resultant intensity at P is

$$|\vec{E}| = E_1 \cos\theta + E_2 \cos\theta$$

$$|\vec{E}| = 2E_1 \cos\theta \qquad (\because |E_1| = |E_2|)$$

$$|\vec{E}| = \frac{2}{4\pi\varepsilon_0} \frac{q}{(r^2+a^2)} \cos\theta$$

$$= \frac{2}{4\pi\varepsilon_0} \frac{q}{(r^2+a^2)} \frac{OA}{AP}$$

$$= \frac{2}{4\pi\varepsilon_0} \frac{q}{(r^2+a^2)} \times \frac{a}{\sqrt{r^2+a^2}}$$

$$= \frac{q(2a)}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}}$$

$$|\vec{E}| = \frac{|\vec{p}|}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}} \quad \text{———} \quad (3)$$

The direction of $\vec{E}$ is along $\vec{PR} \parallel \vec{BA}$ (i.e opposite to $\vec{p}$)

In vector form eq (3) can be written as

$$|\vec{E}| = \frac{-\vec{p}}{4\pi\varepsilon_0 (r^2+a^2)^{3/2}} \quad \text{———} \quad (4)$$

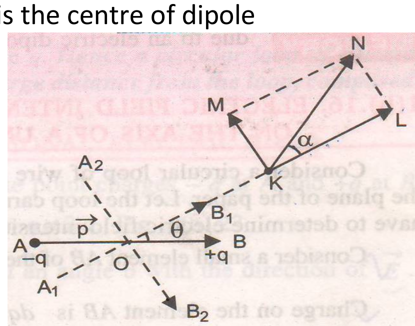
If the dipole is short, $2a << r$

$$\therefore |\vec{E}| = \frac{|\vec{p}|}{4\pi\varepsilon_0 r^3} \quad \text{———} \quad (5)$$

Clearly $|\vec{E}| \propto 1/r^3$

Ratio: $\dfrac{E_{axial}}{E_{equatorial}} = 2$

ELECTRIC FIELD INTENSITY AT ANY POINT DUE TO A SHORT ELECTRIC DIPOLE: - AB represent a short electric dipole of moment **p** along **AB**. O is the centre of dipole



We have to calculate electric field intensity **E** at any point k, where OK = r, <BOK = θ

The dipole moment **p** can be resolved into two rectangular components
$p\cos\theta$ along A_1B_1 and $p\sin\theta$ along $A_2B_2 \perp A_1B_1$
Field intensity at K on the axial line of A_1B_1

$$|\vec{E}_1| = \frac{2p\cos\theta}{4\pi\varepsilon_0 r^3} \quad \text{——— (1)}$$

It is represented by $\vec{KL}$ along OK.

Field intensity at K on equatorial line of A_2B_2

$$|\vec{E}_2| = \frac{p\sin\theta}{4\pi\varepsilon_0 r^3} \quad \text{——— (2)}$$

It is represented by $\vec{KM} \parallel B_2A_2$ & $\perp \vec{KL}$.

In ||gm KLNM.

By ||gm $KN = \sqrt{KL^2 + KM^2}$

$$\therefore |\vec{E}| = \sqrt{E_1^2 + E_2^2}$$

$$= \sqrt{\left(\frac{2p\cos\theta}{4\pi\varepsilon_0 r^3}\right)^2 + \left(\frac{p\sin\theta}{4\pi\varepsilon_0 r^3}\right)^2}$$

$$= \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{4\cos^2\theta + \sin^2\theta}$$

$$|\vec{E}| = \frac{p}{4\pi\varepsilon_0 r^3}\sqrt{3\cos^2\theta + 1}$$

i.e. $$\boxed{|\vec{E}| = \frac{p\sqrt{3\cos^2\theta + 1}}{4\pi\varepsilon_0 r^3}} \quad \text{——— (3)}$$

For Calculation of α

In ΔKLN

$$\tan\alpha = \frac{LN}{KL}$$

$$= \frac{KM}{KL} = \frac{|\vec{E_2}|}{|\vec{E_1}|} \quad \text{(By eq (1) \& (2))}$$

$$= \frac{p \sin\theta}{4\pi\varepsilon_0 r^3} \times \frac{4\pi\varepsilon_0 r^3}{2p\cos\theta}$$

$$= \frac{\sin\theta}{2\cos\theta}$$

$$\boxed{\tan\alpha = \frac{1}{2}\tan\theta} \quad \text{———— (4).}$$

Particular cases:- 1. when the point K lies on axial line of dipole. $\theta = 0°$, $\cos\theta = \cos 0° = 1$

$$\therefore |\vec{E}| = \frac{2p}{4\pi\varepsilon_0 r^3} \quad \text{(By eq (3))}$$

$$\tan\alpha = \frac{1}{2}\tan 0° = 0 \implies \alpha = 0°$$

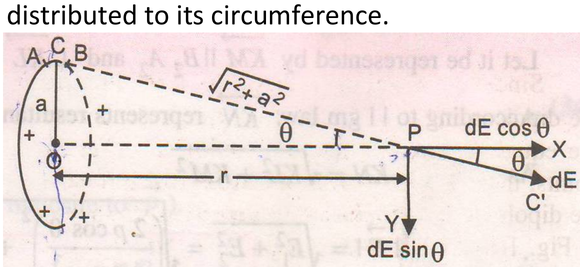
2. when the point K lies on equatorial line of dipole.

$$\theta = 90°, \ \cos\theta = \cos 90° = 0$$

$$\therefore |\vec{E}| = \frac{p}{4\pi\varepsilon_0 r^3} \quad \text{(By eq (3)) and } \tan\alpha = \frac{1}{2}\tan 90°$$

$$\tan\alpha = \infty \implies \alpha = 90°.$$

ELECTRIC FIELD INTENSITY AT ANY POINT ON THE AXIS OF UNIFORMLY CHARGED RING: - Consider a circular loop of wire of negligible thickness, radius a and centre O held perpendicular to the plane of paper. Let the total charge on loop is +q and uniformly distributed to its circumference.



We have determine electric field intensity at point P on the axis of the loop where OP = r

Consider a small element AB of the loop.

Charge on the element AB is dq = $\frac{q\,dl}{2\prod a}$ -------------------------------------- (1)

Electric field intensity at P due to the charge element AB is

$$|d\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{CP^2}$$

$$|d\vec{E}| = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2+a^2} \quad \text{———} (2)$$

d**E** can be resolved into two rectangular components

dEcosθ along PX, the axis of the loop.

dEsinθ along PY, ⊥ to the axis of the loop.

Diagrametically opposite element of the loop, components of electric field intensity perpendicular to the axis will cancel

∴ ∑dEsinθ = 0

Hence the resultant electric field intensity **E** at P

|**E**|= ∑dE cosθ

In ΔOPC, $\cos\theta = \frac{OP}{CP} = \frac{r}{(r^2+a^2)^{1/2}}$

$$\therefore |\vec{E}| = \sum \frac{1}{4\pi\varepsilon_0}\frac{dq}{(r^2+a^2)}\cdot\frac{r}{(r^2+a^2)^{1/2}}$$

By eq (1) $dq = \frac{q\,dl}{2\pi a}$

we get $|\vec{E}| = \sum \frac{1}{4\pi\varepsilon_0}\left(\frac{q\,dl}{2\pi a}\right)\frac{r}{(r^2+a^2)^{3/2}}$

$$= \frac{qr}{4\pi\varepsilon_0 \times 2\pi a\,(r^2+a^2)^{3/2}} \sum_{\text{whole loop}} dl$$

$$= \frac{qr \times \cancel{2\pi a}}{4\pi\varepsilon_0 \times \cancel{2\pi a}\,(r^2+a^2)^{3/2}}$$

$$= \frac{qr}{4\pi\varepsilon_0\,(r^2+a^2)^{3/2}} \quad \text{———} (3)$$

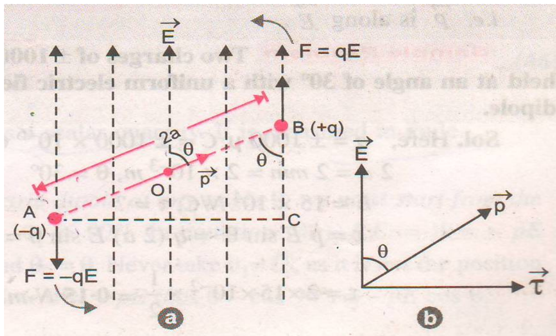
The direction of $\vec{E}$ is along PX, the axis of the loop.

SPECIAL CASES: - (1) when P lies at the centre of the loop, r=0, therefore from equation (3) **E** = 0

(2) When r>> a, then **E** = $qr/4\prod\varepsilon_0 r^3 = q/4\prod\varepsilon_0 r^2$, along PX

Hence a circular loop of charge behaves as a point charge when the observation point P is at very large distance from the loop, compared to radius of the loop.

ELECTRIC DIPOLE IN A UNIFORM TWO DIMENSIONAL ELECTRIC FILED: - consider an electric dipole |**p**| = qX2a be held in uniform external electric field **E** at an angle θ with the direction **E**



Force on charge +q at B = q**E** along the direction of **E**

Force on charge –q at A =q**E** in direction opposite to **E**

Since **E** is uniform so net force qE – qE = 0

If force are equal, unlike & || , acting at different points

They form a couple which rotates the dipole in the anticlock wise direction

⊥distance between the force = arm of couple = AC

As torque = moment of the couple

τ = force X arm of couple

$\tau = F \times AC$

$= F \times AB \sin\theta$

$= (qE)\, 2a \sin\theta$

$\tau = pE \sin\theta$ (p = qX 2a)

In vector form **τ** = **p** X **E**

Direction of **τ** is given by right handed screw rule and is ⊥ to **p** & **E**

SPECIAL CASES (1) θ = 0 τ = pE sin 0 = 0

The dipole is in stable equilibrium

If $\theta = 180^0$, the dipole will be in an unstable equilibrium

(2) When $\theta = 90^0$

Torque is maximum

$\tau = pE \sin 90^0$

$\tau = pE$

POTENTIAL ENERGY OF DIPOLE IN A UNIFORM ELECTRIC FIELD: -

"Potential energy of dipole is the energy possessed by the dipole by virtue of its particular position in the electric field"

The torque **τ** acting on the dipole is τ = pE sinθ

Small amount of work done in rotating the dipole through a small angle dθ against the torque

$dw = \tau d\theta$

$= pE \sin\theta\, d\theta$

Total work done in rotating the dipole from θ_1 to θ_2

$W = \int_{\theta}^{\theta} pE\sin\theta\, d\theta$

$= pE[-\cos\theta]_{Q1}^{Q2}$

$= -\, pE[\cos\theta_2 - \cos\theta_1]$ --(1)

Potential energy of dipole

$U = W = -\, pE[\cos\theta_2 - \cos\theta_1]$ ----------------------------------(2)

PARTICULAR CASES: - (1) when the dipole is initially aligned along the electric field i.e. $\theta_1 = 0^0$ and we have to set it at angle θ with **E** i.e. $\theta_2 = \theta$

By equation (2)

$U = W = - pE[\cos\theta - \cos 0^0]$

$= -pE(\cos\theta - 1)$

(2) When the dipole is initially a right angle to **E** i.e. $\theta_1 = 90^0$ and we have to set it an angle θ with **E** i.e. $\theta_2 = \theta$

By equation (2)

$U = W = - pE[\cos\theta_2 - \cos 90^0]$

$= - pE \cos\theta$

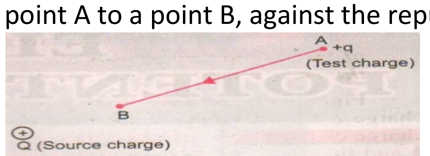
En vector form $U = W = - \mathbf{p}.\mathbf{E}$

Potential energy of an electric dipole is a scalar quantity and measure in joule.

ELECTROSTATIC POTENTIAL AND FLUX: - in the previous chapter, we have learnt the concept of electric field due to a point charge or a distribution of charges. It is represented in terms of a vector quantity **E**, called electric field intensity. In this chapter we shall learn that electric field can also be represented in terms of a scalar quantity V, called electrostatic potential.

Basically, electrostatic potential of a charged body represents the degree of electrification of the body. It determines the direction of flow of chare between two charged bodies placed in contact with each other. The charge always flow from higher potential to lower potential and stop when two bodies have same potential.

ELECTROSTATIC POTENTIAL ENERGY: -let us assume that electrostatic field **E** is due to charge +Q placed at the origin. Let a small test charge +q be brought from a point A to a point B, against the repulsive force on it due to charge +Q.



We shall assume that the test charge +q is so small that it does not disturb the configuration of charge +Q at the origin. We assume that an external force F_{ext} applied is just sufficient to counter the repulsive electric force $\mathbf{F_E}$ on the test charge q so that net force on the test charge q is zero and it moves from A to B without any acceleration.

∴ Work done by external force in moving the test charge +q from A to B is

$$W_{AB} = \int_A^B \boldsymbol{F}_{\mathbf{ext}}.\mathbf{dr} = - \int_A^B \boldsymbol{F}_{\mathbf{E}}.\mathbf{dr} \text{ --(1)}$$

This work done against electrostatic force gets stored as potential energy.

Electrostatic potential energy difference between points B and A

$$\Delta U = U_B - U_A = W_{AB} \text{ --- (2)}$$

Hence we define

Electrostatic potential energy difference between the two points B and A as the minimum work required to be done by an external force in moving without acceleration a test charge q from A to B.

- Work done by an electrostatic field in moving a given charge from one point to another depends only on the positions of initial and final points. It does not depend on the path chosen in going from one point to the other.

For a charge distribution of finite extent, we choose zero electrostatic potential energy at infinite.

We get from equation (2)

$$W_{\infty B} = U_B - U_\infty = U_B - 0 = U_B \text{ ----------------------------(3)}$$

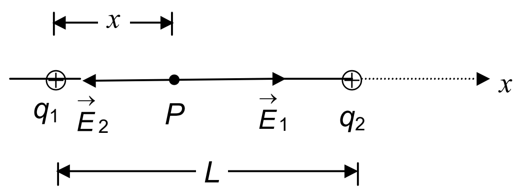
Hence we may define

" electrostatic potential energy of a charge q at a point in an electrostatic field due to any charge congiguration as the work done by the external force (equal and opposite to electric force) in bringing the charge q from infinite to that point any acceleration**"**

Illustration 9

Question: **A point charge q_1 of + 1.5 μC is placed at a origin of a co-ordinate system, and the second charge q_2 of + 2.3 μC is at a position $x = L$, where $L = 13$ cm. At what point P along the x-axis is the electric field zero (in mm).**

Solution: The point must lie between the charges because only in this region the forces exerted by q_1 and q_2 on a test charge oppose each-other. If $\vec{E}_1$ is the electric field due to q_1 and $\vec{E}_2$ is that due to q_2, the magnitudes of these vectors must be equal, or



$$E_1 = E_2$$

We then have

$$\frac{1}{4\pi\varepsilon_0}\frac{q_1}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{(L-x)^2}$$, where x is the co-ordinate of the point P.

Solving for x, we have

$$x = \frac{L}{1 \pm \sqrt{q_2/q_1}}$$

Substituting numerical values for L, q_1 and q_2,
We obtain

$x = 5.8$ cm and $x = -54.6$ cm

But the negative value of x is unacceptable

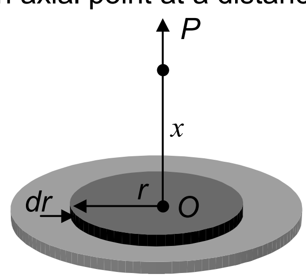
Hence, $x =$ **58 mm**

ELECTRIC FIELD DUE TO A UNIFORMLY CHARGED DISC AT A POINT ON THE AXIS OF THE DISC

Let us consider a flat, circular, non-conducting thin disc of radius R having a uniform surface charge density σ c/m^2. We have to find the electric field intensity at an axial point at a distance x from the disc.

Let O be the centre of a uniformly charged disc of radius R and surface charge density σ. Let P be an axial point, distant x from O, at which electric field intensity is required.

From the circular symmetry of the disc, we imagine the disc to be made up of a large number of concentric circular rings and consider one such ring of radius r and an infinitesimally small width dr



The area of the elemental ring = circumference x width = ($2\pi\, rdr$)

The charge dq on the elemental ring = ($2\pi rdr$) σ

Therefore, the electric field intensity at P due to the elementary ring is given by

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{(2\pi rdr)\sigma x}{(r^2+x^2)^{3/2}} = \frac{\sigma x}{2\varepsilon_0}\frac{rdr}{(r^2+x^2)^{3/2}},$$

and is directed along the x-axis. Hence, the electric intensity E due to the whole disc is given by

$$E = \frac{\sigma x}{2\varepsilon_0}\int_O^R \frac{rdr}{(r^2+x^2)^{3/2}}$$

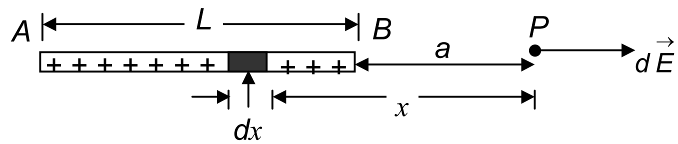
$$= \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{(r^2+x^2)^{1/2}}\right]_0^R = \frac{\sigma x}{2\varepsilon_0}\left[-\frac{1}{(R^2+x^2)^{1/2}} + \frac{1}{x}\right]$$

$$= \frac{\sigma}{2\varepsilon_0}\left[1 - \frac{x}{(R^2 + x^2)^{1/2}}\right] \qquad \ldots \textbf{(9B)}$$

Illustration 10

Question: **A thin insulating rod of length *L* carries a uniformly distributed charge *Q*. Find the electric field strength at a point along its axis a distance '*a*' from one end. (In N/C)**
(take $Q = 10^{-9}$ C, a = 1m, L = 2m)

Solution: Let us consider an infinitesimal element of length dx at a distance x from the point P. The charge on this element is $dq = \lambda dx$, where λ (= $\frac{Q}{L}$) is the linear charge density.



The magnitude of the electric field at P due to this element is

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{x^2} = \frac{1}{4\pi\varepsilon_0}\frac{(\lambda dx)}{x^2}$$

and its direction is to the right since λ is positive. The total electric field strength E is given by

$$E = \frac{1}{4\pi\varepsilon_0}\lambda\int_a^{a+L}\frac{dx}{x^2}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[-\frac{1}{x}\right]_a^{a+L}$$

$$= \frac{\lambda}{4\pi\varepsilon_0}\left[\frac{1}{a} - \frac{1}{a+L}\right]$$

$$= \frac{Q}{(4\pi\varepsilon_0)\, a(a+L)} = \mathbf{3}$$

MOTION OF A CHARGED PARTICLE IN AN UNIFORM ELECTRIC FIELD

A particle of mass m and charge q in an uniform electric field $\vec{E}$ experiences a force

$$\vec{F} = q\vec{E}$$

From Newton's second law of motion,

$$\vec{F} = m\vec{a}$$

Hence, the acceleration of the charged particle in the uniform electric field is

$$\vec{a} = \frac{q\vec{E}}{m}$$

Since the field is uniform, the acceleration is constant in magnitude and direction. So we can use the equation of kinematics for constant acceleration. Now, there are two possibilities.

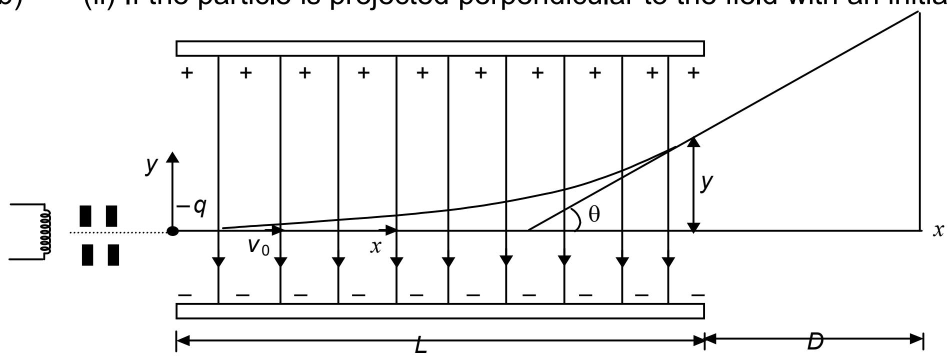
(a) If the particle is initially at rest
From equation $v = u + at$, we get

$$v = at = \frac{qE}{m}t \left[\because u = 0;\ a = \frac{qE}{m}\right]$$

From equation $S = ut + \frac{1}{2}at^2$, we have

$$S = \frac{1}{2}at^2 = \frac{qE}{2m}t^2$$

(b) (ii) If the particle is projected perpendicular to the field with an initial velocity v_0.



For motion along x-axis, we have $v_x = v_0$ = constant ($\because u = v_0$ and $a = 0$)

$\therefore\ x = v_0 t$ … (i)

for motion along y-axis, we have

$$y = \frac{1}{2}\left[\frac{qE}{m}\right]t^2 \quad \text{… (ii)}$$

$$\left[\because u = 0;\ a = \frac{qE}{m}\right]$$

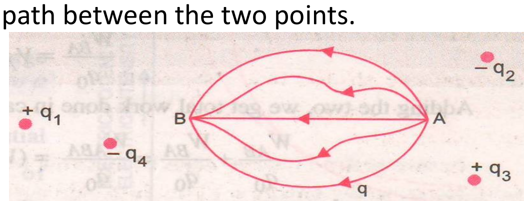
Substituting the value of t from equation (i) in equation (ii), we get

$$y = \frac{qE}{2m}\left[\frac{x}{v_0}\right]^2$$

$$= \frac{qE}{2mv_0^2}x^2$$

Which is the equation of the parabola.

ELECTROSTATIC POTENTIAL: - electric potential difference between two point B and A in an electrostatic field as the amount of work done in carrying unit positive test charge without acceleration from A to B (against the electrostatic force of field) along any path between the two points.



If V_A and V_B are the electrostatic potentials at A and B respectively.
Then from equation (2)

$$(U_B - U_A)/q = W_{AB}/q = V_B - V_A = \Delta V \text{ --------------------------- (4)}$$

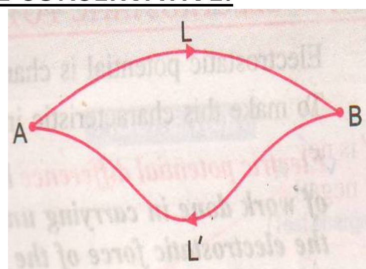
The dimensional formula $[M^1L^2T^{-3}A^{-1}]$

SI unit of electrostatic potential difference is Volt.

1 Volt = 1J/1C

ONE VOLT: - electrostatic potential difference between any two points in an electrostatic field is said to be one volt when one joule of work is done in moving a positive charge of one coulomb from one point to the other against the electrostatic force of the field without any acceleration.

ELECTROSTATIC FORCES ARE CONSERVATIVE: -



Work done in carrying unit positive charge from A to B along length L

$W_{AB}/q_0 = V_B - V_A$ ---------------------------- (1)

Similarly work done in carrying unit positive charge from B to A along any other length L'

$W_{BA}/q_0 = V_A - V_B$ ---------------------------- (2)

Adding eq (1) and (2), we get

$W_{AB}/q_0 + W_{BA}/q_0 = V_B - V_A + V_A - V_B$

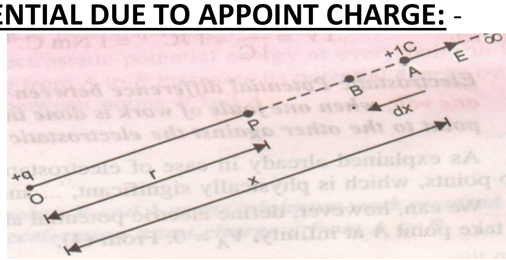
$= 0$

Hence no work is done in moving a unit positive test charge over a closed path in an electric field

Hence electrostatic field is a conservative field and electrostatic forces are conservative forces in nature.

Mathematically $\oint \mathbf{E}\,\mathbf{dl} = 0$

ELECTROSTATIC POTENTIAL DUE TO APPOINT CHARGE: -



Suppose we have calculate electrostatic potential at any point P due to a single point charge +q at O. where OP = r

We know that, electrostatic potential at P is the amount of work done in carrying a unit +ve charge from ∞ to P.

Let A be the intermediate point where OA = x

∴ The electrostatic force on unit positive charge is

$\mathbf{F} = q.1/4\pi\varepsilon_0 x^2$ along OA produced.

Small amount of work done to bring charge from A to B, where AB = dx

$W = \mathbf{F}.\,\mathbf{dx}$

$= Fdx \cos 180^0$

$= - F\,dx$

∴ Total work done in moving unit +ve charge from ∞ to the point P is

$W = \int_{\infty}^{r} -F.dx$

$$= -\int_{\infty}^{r} \frac{1}{4\pi\varepsilon_0}\frac{q}{x^2}\,dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\int_{\infty}^{r} x^{-2}\,dx$$

$$= -\frac{q}{4\pi\varepsilon_0}\left|\frac{x^{-1}}{-1}\right|_{\infty}^{r}$$

$$= \frac{q}{4\pi\varepsilon_0}\left|\frac{1}{x}\right|_{\infty}^{r}$$

$$= \frac{q}{4\pi\varepsilon_0}\left(\frac{1}{r}-\frac{1}{\infty}\right)$$

$$= \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}$$

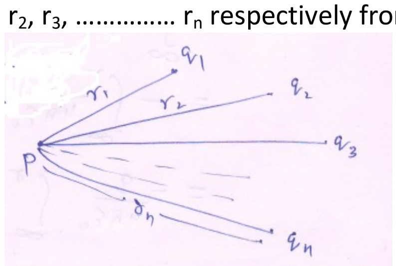
By definition, this is the electrostatic potential at P due to charge q

$V = W/1 = q / 4\pi\varepsilon_0 r$

At $r = \infty$; $V = 0$

i.e. electrostatic potential due to a single charge is spherically symmetric.

POTENTIAL AT A POINT DUE TO GROUP OF ELECTRIC CHARGES: -suppose there are a number of point charges q_1 , q_2, q_3 --------- q_n at distances r_1, r_2, r_3, …………… r_n respectively from point P



Potential at P due to charge q_1

$V_1 = q_1/4\pi\varepsilon_0 r_1$ --------------------------------- (1)

Similiarly Potential at P due to charge $q_2, q_3 --- q_n$.

$$V_2 = \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} \; ; \; V_3 = \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3}$$

$$\vdots$$

$$V_n = \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n} \quad \text{——— (2)}$$

Total Potential at P.

$$V = V_1 + V_2 + V_3 + --- + V_n$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r_1} + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r_2} + \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r_3} + ---- + \frac{1}{4\pi\varepsilon_0}\frac{q_n}{r_n}$$

(By eq (1) & (2))

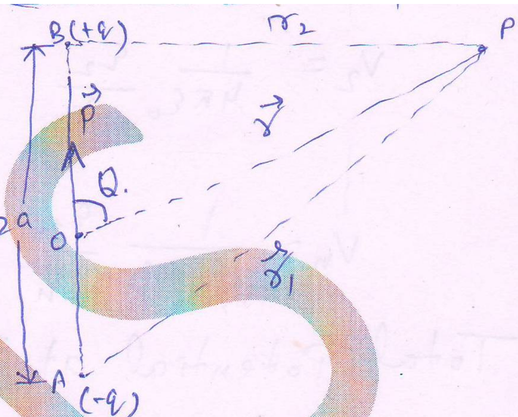
$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1}{r_1} + \frac{q_2}{r_2} + \frac{q_3}{r_3} + ---- + \frac{q_n}{r_n}\right]$$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{r_i}}$$

If $\vec{r}_1, \vec{r}_2, \vec{r}_3, --- \vec{r}_n$ are position vectors of the charges $q_1, q_2, q_3, --- q_n$ resp., then electrostatic potential at point P, whose position vector is $\vec{r}_0$

$$\boxed{V = \frac{1}{4\pi\varepsilon_0}\sum_{i=1}^{n}\frac{q_i}{|\vec{r}_0 - \vec{r}_i|}}$$

ELECTROSTATIC POTENTIAL AT A POINT DUE TO AN ELECTRIC DIPOLE: -



The dipole moment

$$|\vec{p}| = q \times 2a \quad \text{——— (1)}$$

let us take the origin at the centre of the dipole. we have to calculate electric potential at any point P where $\vec{OP} = \vec{r}$ and $\angle BOP = \theta$.

Electrostatic Potential at P due to $-q$ charge at A

$$V_1 = \frac{-q}{4\pi\varepsilon_0 r_1} \quad \text{——— (2)}$$

and Electrostatic Potential at P due to $+q$ charge.

$$V_2 = \frac{q}{4\pi\varepsilon_0 r_2} \quad \text{——— (3)}$$

Potential at P due to the dipole

$$V = V_2 + V_1$$

$$V = \frac{q}{4\pi\varepsilon_0}\left[\frac{1}{r_2} - \frac{1}{r_1}\right] \quad \text{——— (4)}$$

Now by geometry

$$r_1^2 = r^2 + a^2 + 2ar\cos\theta \quad ; \quad r_2^2 = r^2 + a^2 - 2ar\cos\theta$$

$$r_1^2 = r^2\left(1 + \frac{a^2}{r^2} + \frac{2a}{r}\cos\theta\right)$$

If $a << r$, $\frac{a}{r}$ is small, $\frac{a^2}{r^2}$ can be neglected

$$\therefore \quad r_1^2 = r^2\left(1 + \frac{2a}{r}\cos\theta\right)$$

$$r_1 = r\left(1 + \frac{2a}{r}\cos\theta\right)^{+1/2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 + \frac{2a}{r}\cos\theta\right)^{-1/2}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{1}{\cancel{2}} \times \frac{\cancel{2}a}{r}\cos\theta\right) \quad \text{[neglecting higher power]}$$

$$\frac{1}{r_1} = \frac{1}{r}\left(1 - \frac{a}{r}\cos\theta\right)$$

Similarly $\frac{1}{r_2} = \frac{1}{r}\left(1 + \frac{a}{r}\cos\theta\right)$

Sub. $\frac{1}{r_1}$ & $\frac{1}{r_2}$ in eq (4), we have.

$$V = \frac{q}{4\pi\varepsilon_0}\frac{1}{r}\left[1 + \frac{a}{r}\cos\theta - \left(1 - \frac{a}{r}\cos\theta\right)\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\cdot\frac{1}{r}\left[\frac{2a}{r}\cos\theta\right]$$

$$V = \frac{q}{4\pi\varepsilon_0}\cdot\frac{2a\cos\theta}{r^2}$$

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} \qquad [p = q \times 2a] \qquad (5)$$

As $p\cos\theta = \vec{p}\cdot\hat{r}$

$\therefore$ Electrostatic potential at P due to a short dipole $(a << r)$ is

$$\boxed{V = \frac{\vec{p}\cdot\hat{r}}{4\pi\varepsilon_0 r^2}}$$

On the dipole axis. $\theta = 0^\circ$ or π

$$\therefore \quad V = \pm \frac{p}{4\pi\varepsilon_0 r^2}$$

At any point in the equatorial plane.

$\theta = \pi/2$.

$\therefore \quad V = 0 \qquad (\cos\pi/2 = 0)$

i.e electrostatic potential at any point in the equatorial plane of dipole is zero.

EQUIPOTENTIAL SURFACES: -An equipotential surface is that surface at every point of which electric potential is the same.

Potential difference between two points B and A is work done in carrying unit positive test charge from A to B.

$V_B - V_A = W_{AB}$

If points A and B lie on an equipotential surface, then

$V_B = V_A$

$\therefore \quad W_{AB} = V_B - V_A = 0$

No work is done in moving the test charge from one point of equipotential surface to the other.

If dl is the small distance over the equipotential surface through which unit positive charge is carried

Then $dW = \mathbf{E.\ dl} = E\,dl\cos\theta$

$E\,dl\cos\theta = 0$

$\cos\theta = 0$ or $\theta = 90^0$

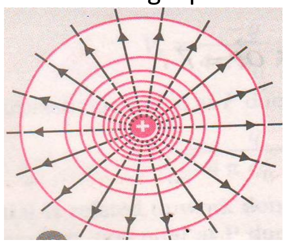
i.e. $\mathbf{E \perp dl}$

i.e. for any charge configuration, equipotential surface through a point is normal to the electric field at that point.

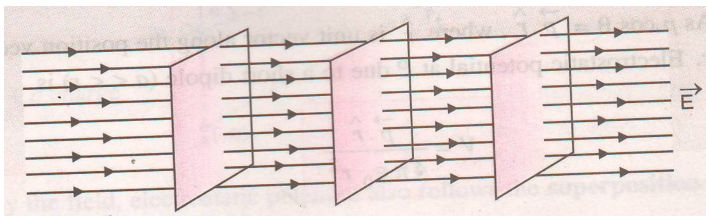
- For a single charge q, the potential $V = q / 4\pi\varepsilon_0 r$

This shows that V is constant if r is constant

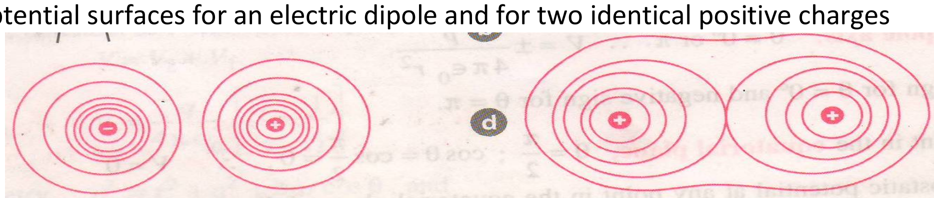
Hence equipotential surfaces of a single point charge are concentric spherical surfaces centred at the charge.



- For a uniform electric field, say along x-axis, the equipotential surfaces are planes normal to the x-axis i.e. planes || to Y-Z plane.



- The equiptential surfaces for an electric dipole and for two identical positive charges



FOR AN ELECTRIC DIPOLE **FOR TWO IDENTICAL POSITIVE CHARGES**

RELATION BETWEEN ELECTRIC INTENSITY AND ELECTRIC POTENTIAL: -

<table>
<tr>
<td>
Let potential of A be $V_A = V$ and potential of B be $V_B = V - dV$
Where dV is decrease in potential in the direction of electric intensity E normal to A and B
Work done $W_{BA} = - E(dr)$
$W_{BA} = V_A - V_B$
$= V - (V - dV)$
$= dV$
$\therefore \quad dV = - E(dr)$
$E = - dV/dr$
Negative sign shows that the direction of electric field E is the direction of decreasing potential.
➤ $E_x = - dV/dx$; $E_y = - dV/dy$; $E_z = - dV/dz$
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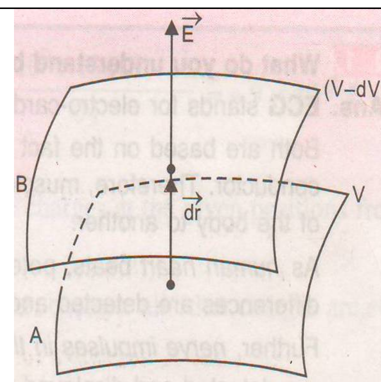

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</table>

POTENTIAL GRADIENT: - The magnitude of electric field is given by change in magnitude of potential per unit displacement normal to the equipotential surface at the point.

$|\mathbf{E}| = - \ |dV|/dr \ = - (\text{ potential gradient})$

- Conclusion concerning the relation between electric field and potential
  (1) Electric field is in the direction in which the potential decreases steepest
  (2) Its magnitude is given by the change in the magnitude of potential per unit displacement normal to the equipotentail surface at the point.

MATA SAVITRI DEVI SANJEEVANI PUBLIC SCHOOL

CLASS- XII ELECTROSTATICS SUBJECT: PHYSICS

BY RUPESH GUPTA SIR

ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF CHARGES: - "electrostatic potential energy of a system of point charges as the total amount of work done in bringing the various changes to their respective positions from infinite large mutual separations"

(a) **ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF TWO POINT CHARGES:** -

Electrostatic potential at P_2 due to charge q_1 at p_1 is

$V = q_1/4\pi\varepsilon_0 r_{12}$

Work done in carrying charge q_2 from ∞ to P_2

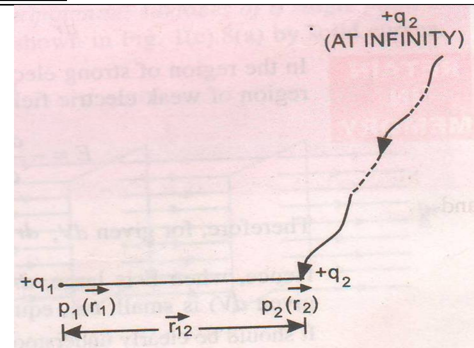
W = potential X charge

$= q_1 \times q_2 / 4\pi\varepsilon_0 r_{12}$

This work done stored in form of potential energy

Thus

$$U = W = \frac{q_1 q_2}{4\pi\epsilon_0 r_{12}}$$



(b) **ELECTROSTATIC POTENTIAL ENERGY OF A SYSTEM OF N POINT CHARGES:** -In bringing the first charge q_1 to position $P_1(\mathbf{r_1})$. No work is done i.e. $w_1 = 0$

When we bring charge q_2 from infinite to $P_2(\mathbf{r_2})$ at a distance r_{12} from q_1 work done

$$W_2 = \frac{q_1}{4\pi\varepsilon_0 r_{12}} q_2$$

In bringing q_3 from ∞ to $P_3(\vec{r}_3)$ work done

$$W_3 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1}{r_{13}} + \frac{q_2}{r_{23}}\right) \times q_3$$

$$= \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right)$$

Similarly $$W_4 = \frac{1}{4\pi\varepsilon_0}\left(\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right)$$

$\therefore$ Electrostatic Potential energy of a system of four charges

$U = W_1 + W_2 + W_3 + W_4$

$$= 0 + \frac{1}{4\pi\varepsilon_0}\frac{q_1 q_2}{r_{12}} + \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}}\right] + \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_2 q_3}{r_{23}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

$$= \frac{1}{4\pi\varepsilon_0}\left[\frac{q_1 q_2}{r_{12}} + \frac{q_1 q_3}{r_{13}} + \frac{q_1 q_4}{r_{14}} + \frac{q_2 q_3}{r_{23}} + \frac{q_2 q_4}{r_{24}} + \frac{q_3 q_4}{r_{34}}\right]$$

In this way we can write electrostatic potential energy of a system of N point charges

$$\boxed{U = \frac{1}{2} \sum_{j=1}^{j=N} \sum_{\substack{k=1 \\ j \neq k}}^{k=N} \frac{1}{4\pi\varepsilon_0} \frac{q_j q_k}{r_{jk}}} \quad \text{—— (1)}$$

$$\boxed{U = \frac{1}{2} \sum_{j=1}^{N} q_j V_j} \quad \text{—— (2)}$$

Equation (2) represents relation between electrostatic potential energy and electrostatic potential

Unit of electrostatic potential energy: - joule

1 electron volt = 1.6X 10^{-19} joule

POTENTIAL ENERGY OF CHARGES IN AN EXTERNAL ELECTRIC FIELD: -

(a) **POTENTIAL ENERGY OF A SINGLE CHARGE IN AN EXTERNAL FIELD:** - The external electric field **E** and the corresponding external potential V may change from point to point. If V(**r**) is external potential at any point P of position vector **r**, then
Work done in bringing a unit positive charge from infinite to the point P is equal to V
Therefore work done in bringing a charge q from infinity to the point P in the external field = qV(**r**)
This work done stored in form of potential energy
∴ Potential energy of a single charge q at **r** in an external field = q. V(**r**)

(b) **POTENTIAL ENERGY OF A SYSTEM OF TWO CHARGES IN AN EXTERNAL FIELD:** - suppose q_1, q_2 are two point charges at position vectors $\mathbf{r_1}$ and $\mathbf{r_2}$ in a uniform external electric field of intensity **E**.
Work done in bringing charge q_1 from infinite to position $\mathbf{r_1}$
$W_1 = q_1.V(\mathbf{r_1})$
Work done in bringing charge q_2 from infinity to position $\mathbf{r_2}$ against external field $W_2 = q_2.V(\mathbf{r_2})$
Against the field due to q_1 $W_3 = q_1q_2/4\pi\varepsilon_0 r_{12}$
Total work done = $W_1 + W_2 + W_3$
P.E (U) = $q_1.V(\mathbf{r_1}) + q_2.V(\mathbf{r_2}) + q_1q_2/4\pi\varepsilon_0 r_{12}$

***Illustration* 11**

Question: **The electric potential at point *A* is 200 V and at *B* is –400 V. Find the work done by an external force and electrostatics force in moving charge of 2×10^{-8} C slowly from *B* to *A*. (in μJ)**

Solution: Here,

$q_0 = 2 \times 10^{-8}$ C; $V_A = 200\ V$;
$V_B = -400\ V$

work done by the external force = $W_{B\to A}$
$= q_0 (V_A - V_B)$
$= (2 \times 10^{-8}) [(200 - (-400)]$

Work done by the electric force = $-(W_{B\to A})_{\text{external}}$
= **12**

***Illustration* 12**

Question: **Find the work done by some external force in moving a charge q = 2 μC from infinity to a point where electric potential is 10^6V. (in J)**

Solution: $(E_{\infty \to A})_{\text{external}} = (2 \times 10^{-6})(10^4)$

$= \mathbf{2}$

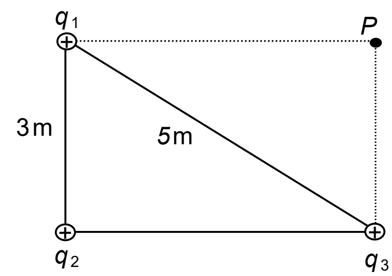


***Illustration* 13**

Question: **Three point charges q_1 = 1μC; q_2 = 2 μC; and q_3 = 3 μC are fixed at a position shown. How much work would be needed to bring a charge q_4 = 25 μC from infinity and to place it at *P?* (in mJ)**

Solution: The external work is $W_{\text{ext}} = q[V_f - V_i]$

In this case, $V_i = 0$.

So, $W_{\text{ext}} = q_4 V_P = (2.5 \times 10^{-6}\text{ C})(7.65 \times 10^3\text{ V})$ =**19**

***Illustration* 14**

Question: **The electrical potential due to a point charge is given by $V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r}$. Find the x-component of the electric field .(take r = 2m, x = 1m, Q = $\frac{8}{9} \times 10^{-9}$ C) (in N/C)**

Solution: (a) The radial component is given by

$$E_r = -\frac{dV}{dr} = +\frac{1}{4\pi\varepsilon_0}\frac{Q}{r^2}$$

(b) In terms of rectangular components, the radial distance is $r = (x^2 + y^2 + z^2)^{1/2}$; therefore, the potential function

$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{r} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{(x^2+y^2+z^2)^{1/2}}$$

To find the x-component of the electric field, we treat y and z constants. Thus

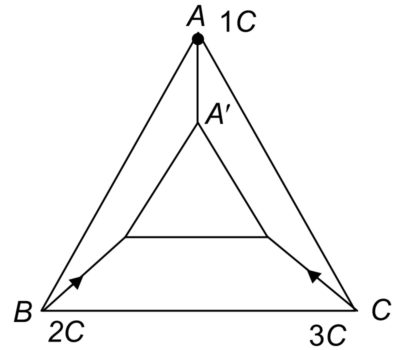
$$E_x = -\frac{\partial V}{\partial x}$$

or $$E_x = +\frac{1}{4\pi\varepsilon_0}\frac{Qx}{(x^2+y^2+z^2)^{3/2}}$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{Qx}{r^3} = \mathbf{1}$$

***Illustration* 15**

Question: **Three point charges 1μC, 2μC and 3μC are placed at the corner of an equilateral triangle of side 1m. Calculate the work required to move these charges to the corners of a smaller equilateral triangle of side 0.5 m as shown. (in mJ)**



Solution: As the potential energy of two point charges separated by a distance 'r' is given by [$U = \frac{q_1 q_2}{4\pi\varepsilon_0 r}$], the initial and the final potential energy of the system will be

$$U_I = \frac{10^{-12}}{4\pi\varepsilon_0}\left[\frac{1\times 2}{1}+\frac{2\times 3}{1}+\frac{3\times 1}{1}\right]$$

$$= 9 \times 10^9 \times 11 \qquad = 9.9 \times 10^{-2}\text{ J}$$

$$U_F = \frac{10^{-12}}{4\pi\varepsilon_0}\left[\frac{1\times 2}{0.5}+\frac{2\times 3}{0.5}+\frac{3\times 1}{0.5}\right] = 9\times 10^9 \times 22\times 10^{-12}$$

$$= 19.8 \times 10^{-2}\text{ J}$$

So, the work done in changing the configuration of the system

$$W = U_F - U_I = (19.8 - 9.9) \times 10^{10} \times 10^{-12}$$

$$= \mathbf{99\ mJ}$$

Illustration 16

Question: **A dipole whose dipole moment is *p* lies along the x-axis $(\vec{p} = p\hat{i})$ in a non-uniform field $\vec{E} = \frac{C}{x}\hat{i}$. What is the magnitude of the force on the dipole? (take, *P* = 2 SI unit, *C* = 1 SI unit, x = 1 SI unit) (in SI unit)**

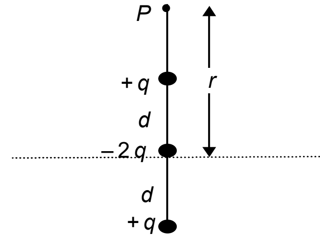
Solution: We have $U = -\vec{p}.\vec{E} = -p\hat{i}.\frac{C}{x}\hat{i} = -\frac{pC}{x}$

Now $F = -\frac{dU}{dx} = -\frac{d}{dx}(-\frac{pC}{x}) = -\frac{pC}{x^2}$ $\Rightarrow$ $\vec{F} = -\frac{pC}{x^2}\hat{i}$ **= 2**

Illustration 17

Question: **An Electric quadrupole consists of two equal and opposite dipoles so arranged that their electric effects do not quite cancel each-other at distant points. In the figure given, Calculate electric potential *V* for the point *P* on the axis of this quadrupole.**

(take $q = \frac{8}{9}\times 10^{-9}$ C, *d* = 1, *r* = 2)



Solution: The electric potential at *P* is given by

$$V = \frac{1}{4\pi\varepsilon_0}\left[\frac{q}{r-d} + \frac{-2q}{r} + \frac{q}{r+d}\right] = \frac{1}{4\pi\varepsilon_0}\ \frac{2qd^2}{r(r^2-d^2)} = \frac{1}{4\pi\varepsilon_0}\ \frac{2qd^2}{r^3\left[1-\frac{d^2}{r^2}\right]}$$

Because $d << r$, we can neglect $\frac{d^2}{r^2}$ compared to 1, in which case the potential becomes

$$V = \frac{1}{4\pi\varepsilon_0}\ \frac{2qd^2}{r^3}\ \mathbf{= 2}$$

AREA VECTOR: - The direction of a planar area vector is along its normal, but a normal can point in two directions, in wards or outwards. By convention, the vector associated with every area element of a closed surface is taken to be in the direction of the outward normal.
An area element vector $\Delta\vec{s}$ at point on a closed surface can be written as

$\Delta\vec{s}$ **= n** Δs ------------------------------ (1)

Δs is the magnitude of area element and **n** is a unit vector in the direction of outward drawn normal at that point.